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Vortex pairs in two-dimensional superconductors

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Abstract. We study classical and quantum mechanics of vortex–vortex and vortex–antivortex pairs in two-dimensional superconductors. The possibility of observing quantum states of pancake vortices in high-temperature superconductors is discussed.

1. Introduction

The dynamics of the magnetic flux in superconductors is of great interest for both fundamental physics and applications. The hopping of vortices between pinning centres and the annihilation of vortex–antivortex pairs in conventional superconductors has been beautifully visualized by the electron holography methods [1]. These experiments raise questions about the trajectory of the vortex pair and the characteristic time of the annihilation. Our work is in part motivated by the desire to give a rigorous mathematical description of these processes. Another part of our motivation comes from the observation [2–15] that vortices in high-temperature superconductors can behave as quantum objects. If one recalls that a vortex is formed by the coherent motion of a macroscopic number of electrons, this seems quite exotic. Nevertheless, a number of experiments on low-temperature magnetic relaxation [2–7] have been interpreted in terms of vortices tunnelling through pinning barriers. Based upon experimental data, it has been also argued that quantum fluctuations of pancake vortices contribute to the thermodynamics [8] and magnetization [9] of high-T_c materials. It is, therefore, not out of the question to look for quantum levels of vortex pairs. If found in experiment, they would provide the most direct evidence of quantum behaviour of vortices.

In this paper we shall limit our consideration to two dimensions (2D), where the vortices behave as particles interacting via a $\ln r$ potential. The problem of two interacting flux lines is of a different level of complexity and will not be addressed here. As far as the classical dynamics is concerned, our results can be applied to thin films of conventional superconductors and to pancake vortices of CuO_2 layers of high-T_c materials. The minimal separation at which the picture of independent vortices makes sense is the size of the vortex core, that is, the coherence length ξ. For CuO_2 layers this picture also breaks down above some critical separation when the interaction between pancake vortices belonging to different planes becomes important. For extremely anisotropic crystals, however, e.g., Bi- or Tl-based copper oxides, a pair of interacting 2D vortices belonging to one plane

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should be a good approximation. Note that the values of ξ in the directions perpendicular and parallel to CuO_2 layers are quite different. In this paper, ξ should be everywhere understood as the coherence length in the CuO_2 plane (which is usually denoted as ξ_{ab}). In high-T_c superconductors it is of the order of a few lattice spacings. Because of that the pancake vortices are really microscopic in nature. Even vortices of that size, however, are formed by the coherent motion of a large number of electrons. Thus, any quantum physics associated with such an object must be of fundamental interest.

To consider the classical dynamics of vortices one should account for all forces acting on them. These are the force of their interaction, the friction force, and the Magnus force. The motion of the vortex–vortex pair is different from the motion of the vortex–antivortex pair. The solutions are also different for free vortices and for the case when the position of one of the vortices is fixed by a strong pinning centre. The important parameter is the ratio of the friction force to the Magnus force. Both are proportional to the velocity of the vortex, so that the ratio is a constant which depends on the material and temperature. The motion of the pair is different in the overdamped regime and in the superclean (or Hall) regime, when the Magnus force dominates. For quantum dynamics of vortices the superclean regime is of major interest. As will be shown below, there are well defined energy levels of vortex pairs in this regime. The strength of the Magnus force, as compared to the force of friction on the moving vortex, is determined by the ratio of the transverse and longitudinal resistivity. In conventional superconductors it is small, indicating that the motion of vortices is dominated by friction. However, both theory [16] and experiment [17] suggest that this ratio grows dramatically as temperature is lowered. Dissipation of the vortex motion is due to the scattering of quasiparticles within the vortex core. At low temperature, the finite spacing of the core levels can strongly reduce dissipation and make the Magnus force the main force in the problem. This should happen [16] when the mean free path of a quasiparticle becomes large compared to $(\epsilon_F/\Delta)\xi$, where ϵ_F is the Fermi energy, and Δ is the superconducting gap. Due to small ϵ_F, large Δ, and small ξ, high-T_c superconductors are likely to enter the superclean regime at $T \ll T_c$ [17, 18].

The paper is organized as follows. The classical motion of vortex–vortex pairs is studied in section 2. Section 3 deals with the classical mechanics of vortex–antivortex pairs. Quantum levels of the pairs are considered in section 4. The possibility of observing the effects studied in this paper is discussed in section 5.

2. Classical mechanics of vortex–vortex pairs

2.1. Free vortices

Consider a thin superconducting layer of thickness d, normal to the z direction. We shall start with a problem of two classical vortices, in the absence of pinning and macroscopic current, separated by a distance $r = |r_1 - r_2|$. At r large compared to the correlation length ξ and up to some limiting value r_{max}, the vortices repel each other via the potential

$$U = -\frac{\Phi_0^2 d}{8\pi^2\lambda^2}\ln\left(\frac{r}{\xi}\right) \tag{1}$$

where $\lambda = (m_e c^2/4\pi e^2 n_s)^{1/2}$ is the bulk penetration length (n_s being the bulk concentration of superconducting electrons) and Φ_0 is the flux quantum. For a thin film, $r_{max} = \lambda^2/d$, while for a layered superconductor it is determined by the strength of the interlayer coupling.

The vortices are also subject to the Magnus force [19–22],

$$F_i = \frac{n_s e d}{c} \dot{r}_i \times \Phi_0 \tag{2}$$

and the force of friction,

$$f_i = -\eta d \dot{r}_i. \tag{3}$$

Here $\Phi_0 = (ch/2e)\hat{z}$ is the vector normal to the layer whose length is the flux quantum, and $i = 1, 2$ numerates the vortices. Formulas (1)–(3) should also apply to pancake vortices in the CuO_2 planes of high-T_c superconductors. In this case m_e should be understood as the effective mass of an electron in the CuO_2 plane, usually denoted as m_{ab}. The presence in these formulas of a loosely defined parameter d (the 'length' of the vortex) should not concern us here, since d enters only through the combination $n_s d$ which is a two-dimensional 'sheet' concentration of electrons.

The inertial mass of the vortex, m_v, is of the order of $m_e k_F d$, where k_F is the Fermi momentum of electrons. We shall assume that m_v can be neglected and will check this later. Then the equations of motion become the conditions that the sum of all forces on each vortex is zero. This gives

$$\begin{aligned} &\frac{n_s e}{c} \dot{r}_1 \times \Phi_0 + \eta \dot{r}_1 - \frac{\Phi_0^2}{8\pi^2\lambda^2} \frac{r_1 - r_2}{(r_1 - r_2)^2} = 0 \\ &\frac{n_s e}{c} \dot{r}_2 \times \Phi_0 + \eta \dot{r}_2 + \frac{\Phi_0^2}{8\pi^2\lambda^2} \frac{r_1 - r_2}{(r_1 - r_2)^2} = 0 \end{aligned} \tag{4}$$

The convenient variables are the radius vector connecting the vortices and the coordinate of the centre of mass of the pair,

$$r = r_1 - r_2 \qquad R = \frac{r_1 + r_2}{2}. \tag{5}$$

In terms of these variables, equations (4) produce two independent equations for r and R:

$$\frac{n_s e}{c} \dot{r} \times \Phi_0 + \eta \dot{r} - \frac{\Phi_0^2}{4\pi^2\lambda^2} \frac{r}{r^2} = 0 \qquad \frac{n_s e}{c} \dot{R} \times \Phi_0 + \eta \dot{R} = 0. \tag{6}$$

The second of these equations implies that the centre of mass of the two vortices remains at rest, $\dot{R} = 0$. Writing the first equation in polar coordinates, r and ϕ, one finds the solution:

$$r^2(t) = r_0^2 + \frac{2\hbar\gamma t}{m_e(1+\gamma^2)} \qquad \phi(t) = \frac{1}{\gamma} \ln \frac{r(t)}{r_0}. \tag{7}$$

Here r_0 is the arbitrary initial separation of vortices and $\gamma = \eta/\pi\hbar n_s$ is the dimensionless strength of friction relative to the Magnus force. Thus, two classical vortices orbit each other, while the distance between them is growing with time due to friction (figure 1). The relative speed goes down with the distance between the vortices as

$$v = \frac{\hbar}{m_e r (1+\gamma^2)^{1/2}}. \tag{8}$$

One way to estimate γ is to use the Bardeen–Stephen formula for η, $\eta = \Phi_0 H_{c2}/\rho_n c^2$, where ρ_n is the normal state resistivity. Choosing the latter in the form $\rho_n = m_e/e^2 n_s \tau$, with τ being the quasiparticle's scattering time, one obtains

$$\gamma = \frac{\hbar\tau}{2m_e\xi^2}. \tag{9}$$

When deriving this formula, we have also used the relation $H_{c2} = \Phi_0/2\pi\xi^2$. Note that both limits $\gamma \ll 1$ and $\gamma \gg 1$ can be obtained for realistic parameter values.

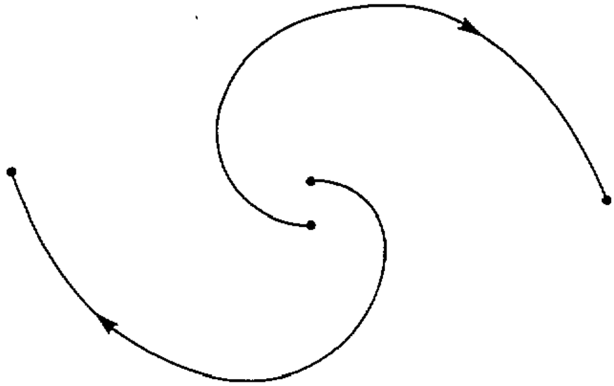

Figure 1. Classical motion of the vortex–vortex pair.

Let us now go back to the question of the inertial mass of the vortex. Comparing the Magnus force with the centripetal force due to inertia, m_v^2/r, one finds that the latter can be neglected if r is large compared to the Fermi wavelength of electrons, $\lambda_F = 2\pi/k_F$, that is, for any physically meaningful separation of vortices.

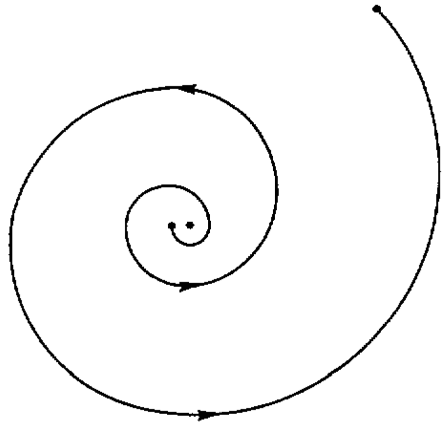

Figure 2. Classical motion of a vortex in the field of a pinned vortex.

2.2. Vortex in the field of a pinned vortex

We shall assume now that the position of one of the vortices is fixed by strong pining. We shall also assume that the pinning potential is short ranged and does not stretch beyond the core of the pinned vortex. In this case, the pinning does not affect the free vortex and the dynamics of the pair reduces to the equation of motion for a free vortex in the field of a pinned vortex,

$$\frac{n_s e}{c}\dot{r}\times\Phi_0 + \eta\dot{r} - \frac{\Phi_0^2}{4\pi^2\lambda^2}\frac{r}{r^2} = 0. \tag{10}$$

The solution of this equation shows that the vortex is orbiting the pinning centre at a speed

$$v = \frac{\hbar}{2m_e r(1+\gamma^2)^{1/2}} \tag{11}$$

with the distance from the centre, r, growing as

$$r^2(t) = r_0^2 + \frac{\hbar\gamma t}{m_e(1+\gamma^2)}. \tag{12}$$

One may notice the difference by a factor of two from the equivalent formulas for free vortices. This motion is illustrated in figure 2.

3. Classical mechanics of vortex–antivortex pairs

3.1. Free pair

We shall now turn to the classical mechanics of the vortex–antivortex pair. Let us begin with a free pair. From the mathematical point of view, the difference from the vortex–vortex pair appears in the sign of the logarithmic interaction; vortex and antivortex attract each other. The sign of the Magnus force also depends on the direction of the magnetic flux and is different for the vortex and the antivortex. Correspondingly,

$$U = \frac{\Phi_0^2 d}{8\pi^2\lambda^2} \ln\left(\frac{r}{\xi}\right) \tag{13}$$

and equations (4) must be rewritten as

$$\begin{aligned} &\frac{n_s e}{c}\dot{\boldsymbol{r}}_1 \times \boldsymbol{\Phi}_0 + \eta\dot{\boldsymbol{r}}_1 + \frac{\Phi_0^2}{8\pi^2\lambda^2}\frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2} = 0 \\ &-\frac{n_s e}{c}\dot{\boldsymbol{r}}_2 \times \boldsymbol{\Phi}_0 + \eta\dot{\boldsymbol{r}}_2 - \frac{\Phi_0^2}{8\pi^2\lambda^2}\frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2} = 0 \end{aligned} \tag{14}$$

where we have preserved the orientation of $\boldsymbol{\Phi}_0$ in the positive z direction. In terms of $\boldsymbol{r}$, $\boldsymbol{R}$, and γ, these equations are equivalent to

$$\dot{\boldsymbol{r}} + \frac{\hbar\gamma}{m_e(1+\gamma^2)}\frac{\boldsymbol{r}}{r^2} = 0 \qquad \dot{\boldsymbol{R}} + \frac{1}{2\gamma}\dot{\boldsymbol{r}} \times \frac{\boldsymbol{\Phi}_0}{\Phi_0} = 0. \tag{15}$$

Equations (15) suggest that the radius vector $\boldsymbol{r}$, connecting the pair, preserves its orientation in space, while the centre of mass is moving in the direction normal to $\boldsymbol{r}$ at a speed

$$V = \frac{\hbar}{2m_e r(1+\gamma^2)}. \tag{16}$$

The separation decreases according to

$$r^2(t) = r_0^2 - \frac{2\hbar\gamma t}{m_e(1+\gamma^2)}. \tag{17}$$

The trajectory of the pair is shown in figure 3. In the limit of zero friction ($\gamma = 0$) the separation of the pair is constant; the vortex and antivortex move parallel to each other at a speed $V = \hbar/2m_e r$, which is uniquely determined by the separation. With a non-zero friction, trajectories of the vortex and antivortex are non-parallel straight lines which intersect at some finite distance. (17) allows us to obtain the annihilation time

$$t_a = \frac{m_e r_0^2}{2\hbar}\left(\gamma + \frac{1}{\gamma}\right). \tag{18}$$

It has a minimum, $t_a = m_e r_0^2/\hbar$, at $\gamma = 1$. This minimal value is of the order of 10^{-12} s for a 100 Å separation, and 10^{-10} s for a 1000 Å separation. The time needed for the annihilation increases in both underdamped ($\gamma \ll 1$) and overdamped ($\gamma \gg 1$) limits. This is easy to understand if one notices that at small γ the pair moves a long way to the point of annihilation, while at large γ the vortex displacement is small, but the motion is slow.

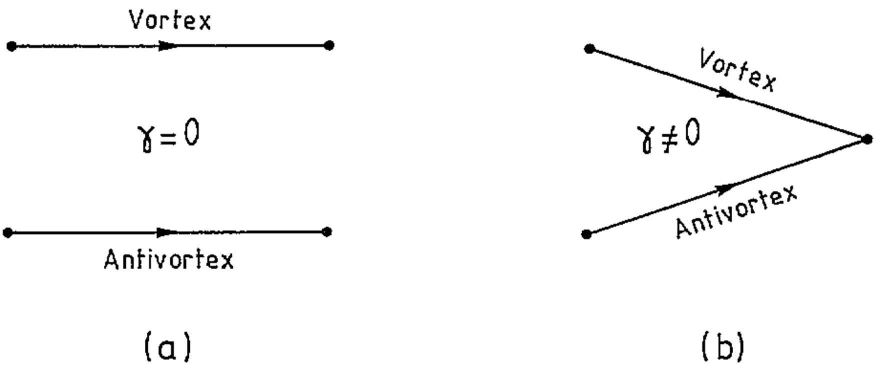

Figure 3. Clasiccal motion of the vortex–antivortex pair.

3.2. *Antivortex in the field of a pinned vortex*

This problem is similar to that for the vortex in the field of a pinned vortex. The only difference is in the sign of the interaction. Consequently, the equation of motion is

$$\frac{n_s e}{c}\dot{r}\times\Phi_0 + \eta\dot{r} + \frac{\Phi_0^2}{4\pi^2\lambda^2}\frac{r}{r^2} = 0. \tag{19}$$

The solution in polar coordinates is

$$r^2(t) = r_0^2 - \frac{\hbar\gamma t}{m_e(1+\gamma^2)} \qquad \phi(t) = \frac{1}{\gamma}\ln\frac{r(t)}{r_0} \tag{20}$$

with the same equation (11) for the orbital speed. The antivortex approaches the pinned vortex on a spiral trajectory shown in figure 4. The annihilation time, $t_a = (m_e r_0^2/\hbar)(\gamma + \gamma^{-1})$, is two times longer than for a free pair.

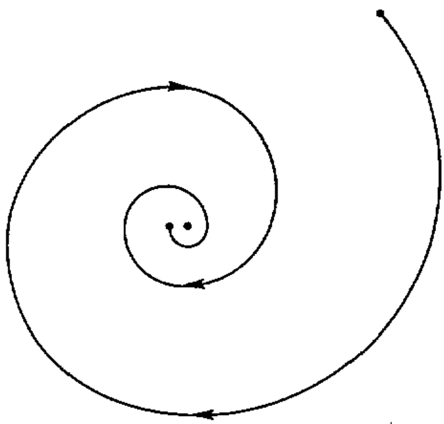

Figure 4. Classical motion of an antivortex in the field of a pinned vortex.

4. Quantum mechanics of vortex pairs

Quantum levels of the vortex pair make sense only in the limit of a very weak friction. This limit corresponds to a superclean regime which high-T_c superconductors are likely to enter at low temperatures (see the introduction). In this section we shall concentrate on a one-body problem of a free vortex (or antivortex) in the field of a pinned vortex. The quantum mechanics of our model in the absence of friction can be obtained if one notices that the effect of the Magnus force on a vortex is equivalent to the effect of a constant magnetic field,

$$B_{eff} = \frac{\pi \hbar n_s d c}{e} \hat{z} \tag{21}$$

on a particle of charge e. Thus, the Hamiltonian of a vortex (antivortex) in the field of a pinned vortex can be written as

$$H = \frac{1}{2m_v} \left[-i\hbar \nabla_1 \pm \frac{e}{c} A_{eff}(r_1) \right]^2 \pm \frac{\Phi_0^2 d}{8\pi^2 \lambda^2} \ln \left(\frac{r}{\xi} \right). \tag{22}$$

Here the sign must be taken as minus for a vortex and plus for an antivortex; the effective vector potential is given by

$$A_{eff} = -\tfrac{1}{2} r \times B_{eff}. \tag{23}$$

If one drops the second term in equation (22), the problem becomes the one of a single free vortex (antivortex) subject to a Magnus force. This problem is equivalent to the problem of a free electron (positron) in the magnetic field. The eigenvalues of the 'kinetic' term are uniformly spaced in energy by $\hbar\omega_{eff}$, where $\omega_{eff} = eB_{eff}/m_v c$ is the effective cyclotron frequency. The limit of $m_v \to 0$ is equivalent to a very high field in the problem of electron. Consequently, the value of $\hbar\omega_{eff}$ can easily be of the order of the Fermi energy. This means that for the problem of a free vortex (antivortex) only the lowest Landau level (LLL) is of interest. The corresponding wave functions are degenerate eigenfunctions of the angular momentum,

$$z^m \exp(-|z|^2/4l^2) \tag{24}$$

with $m = 0, 1, 2, \ldots$, where we have introduced $z = x + iy$ and the effective 'magnetic' length,

$$l^2 = \frac{\hbar c}{e B_{eff}} = \frac{1}{\pi n_s d}. \tag{25}$$

For a monolayer film, πl^2 is the area per electron; l, therefore, is the smallest length in the problem.

Consider now the full Hamiltonian (22). This problem, though for a different potential, is again well known from the problem of an electron in a strong magnetic field. Since the interaction depends only on r, the wave functions again can be chosen as eigenfunctions of the total angular momentum equation (24). As in the previous problem, the additional quantum number, that numerates Landau levels, should be taken at its lowest value because of a huge energy gap between the states evolving from different Landau levels. The procedure called 'the projection of the potential onto the LLL' [23] is briefly outlined below. Let $f(z)$ be the polynomial part of the wave function, $\psi(z) = f(z)\exp(-|z|^2/4l^2)$, and the potential can be expressed as $V(z^*, z)$. Then the eigenvalues of the system can be found from the projected form of the time-independent Schrödinger equation,

$$V\left(2\frac{\partial}{\partial z}, z\right) f(z) = E f(z) \tag{26}$$

where the potential is considered as a power series in which all factors of z^* are placed to the left of all z and then replaced by $2\,\partial/\partial z$.

Let us now apply this procedure to $f(z) = z^m$ and

$$V = V\left(\frac{r^2}{2}\right) = \sum_n C_n 2^{-n} |z|^{2n}. \tag{27}$$

Substituting these functions into equation (26) and performing the differentiation over z, we obtain for the eigenvalues

$$E_m = \frac{1}{m!} \sum_n C_n (n+m)!. \tag{28}$$

Further progress can be made with the help of the formula

$$(n+m)! = \int_0^\infty \mathrm{d}x\, x^{n+m}\, \mathrm{e}^{-x}. \tag{29}$$

Its substitution into equation (28), with account of equation (27), yields

$$E_m = \int_0^\infty \mathrm{d}x\, \frac{x^m}{m!}\, \mathrm{e}^{-x} V(x) \tag{30}$$

where we have chosen $x = r^2/2l^2$.

Equation (30) provides the general expression for the energy levels of the vortex (antivortex) in an arbitrary potential $V(r)$. According to equation (24), the size of the orbit corresponding to the quantum number m is given by

$$\langle r^2 \rangle = \langle |z|^2 \rangle = 2(m+1)l^2. \tag{31}$$

Our approximation holds only for orbits which are much greater than the size of the vortex core, ξ. Recalling that l is the smallest size in the problem, it is clear that only large m make sense from the physical point of view. In this case equation (30) can be simplified. Applying the Stirling formula to $m!$,

$$m! = m^m\, \mathrm{e}^{-m} (2\pi m)^{1/2} \tag{32}$$

we get

$$E_m = \frac{1}{\sqrt{2\pi m}} \int_0^\infty \mathrm{d}x\, \left(\frac{x}{m}\right)^m \mathrm{e}^{m-x} V(x). \tag{33}$$

Let us now choose the new variable of integration, δ, which is related to x by $x = m(1+\delta)$. Then equation (33) becomes

$$E_m = \left(\frac{m}{2\pi}\right)^{1/2} \int_{-1}^\infty \mathrm{d}\delta\ \mathrm{e}^{-m[\delta - \ln(1+\delta)]} V[m(1+\delta)]. \tag{34}$$

We shall assume that the characteristic scale of the potential is much bigger than l. Then it is easy to see that only small δ contribute to the integral. Pulling $V(m)$ out of the integral and expanding the logarithm in the exponent we obtain

$$E_m = \left(\frac{m}{2\pi}\right)^{1/2} V(m) \int_{-1}^\infty \mathrm{d}\delta\ \mathrm{e}^{-m\delta^2/2}. \tag{35}$$

At large m, $-\sqrt{m} < \delta < \sqrt{m}$ effectively contribute to the integral. Consequently, the lower limit of integration can be safely changed to $-\infty$. This yields the trivial answer for large m

$$E_m = V(r = l\sqrt{2m}). \tag{36}$$

Formula (36) can be easily understood from the quasiclassical point of view. In the absence of friction, equations of motion for a vortex subject to the Magnus force and an arbitrary potential V can be written in the form

$$\pi\hbar n_s\,\mathrm{d}\dot{x} = \frac{\partial V}{\partial y}$$
$$-\pi\hbar n_s\,\mathrm{d}\dot{y} = \frac{\partial V}{\partial x} \tag{37}$$

where x and y are components of $\boldsymbol{r}$. These equations are equivalent to the Hamiltonian equations of motion [13, 14], $\dot{q} = \partial V/\partial p$, $\dot{p} = -\partial V/\partial q$, if one chooses

$$q = x \qquad p = \pi\hbar n_s\,\mathrm{d}y. \tag{38}$$

Thus, x and y play the role of generalized coordinate and momentum for the vortex. The next step is to write the quasiclassical quantization condition:

$$\oint p\,\mathrm{d}q = 2\pi\hbar m. \tag{39}$$

Substituting here q and p from equation (38) and integrating over a circular orbit of radius r, we obtain

$$\frac{r_m^2}{2l^2} = m. \tag{40}$$

This relation must hold for large quasiclassical orbits, that is, for large m. From here one immediately obtains the expression (36) for the energy levels.

Going back to our problem of the vortex (antivortex) in the field of a pinned vortex, the question of interest is the distance between the mth and the $(m+1)$th levels. For the logarithmic potential of equation (1) it is given by

$$\Delta E_m = \frac{E_0}{m} \tag{41}$$

where

$$E_0 = \frac{\hbar^2}{4m_e l^2}. \tag{42}$$

Quantum levels of the vortex and the antivortex in the field of a pinned vortex are schematically shown in figure 5.

In the above discussion of quantum dynamics we have completely neglected friction, that is, the interaction of vortices with the dissipative background. A rigorous formulation of this problem is quite difficult since the dissipation of the vortex motion is poorly understood even at the classical level. Meantime, the observation of quantum levels is only possible if their width is small compared to the distance between the levels. One way to estimate the linewidth for the levels of a vortex–antivortex pair is to assume that for quasiclassical orbits it cannot exceed the inverse classical time, Δt_m, of the transition between the mth and the $(m\pm1)$th orbits, which is finite due to friction. According to equations (12), (20), and (40)–(42), this time equals

$$\Delta t_m = \frac{\hbar(1+\gamma^2)}{2\gamma E_0}. \tag{43}$$

Defining the relative linewidth as $\Gamma_m = \hbar/\Delta E_m\,\Delta t_m$, we obtain

$$\Gamma_m = \frac{2\gamma m}{1+\gamma^2}. \tag{44}$$

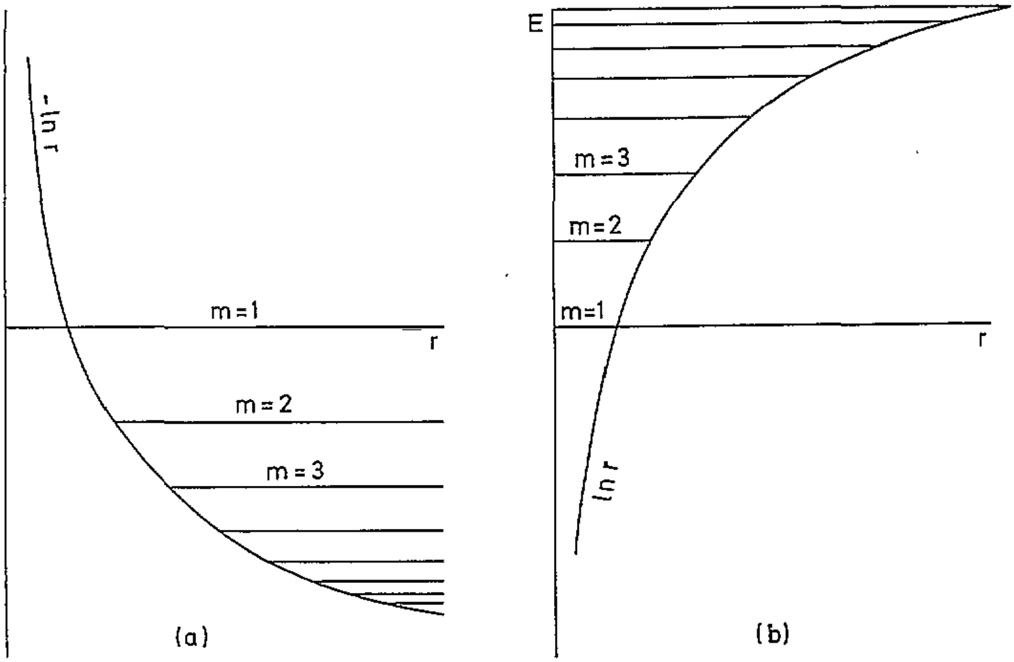

Figure 5. Schematic picture of quantum levels: (a) vortex in the $\ln r$ potential, (b) antivortex in the $\ln r$ potential.

Thus, the relative linewidth is likely to be small for small γ, $\gamma \ll 1/m$. This condition is based upon the classical picture of the dissipation. Due to the quantization of levels of normal electrons inside the vortex core the real condition can be softer.

5. Discussion

The results of previous sections are formally valid for strictly two-dimensional superconductors, that is, for very thin films or monatomic layers. Let us now discuss the possibility of the observation of quantum levels of pancake vortices in high-T_c superconductors. The approximation employed in this paper is based upon logarithmic interaction between the pancakes within one CuO_2 layer; the interaction between vortices belonging to different layers has been neglected. Meantime, when considering large distances, this interaction results in the formation of a vortex line or a closed vortex loop. It has been demonstrated [24–28] that, due to the Josephson interaction between the layers, at $r \sim d(M_e/m_e)^{1/2}$ (where M_e and m_e are electron effective masses in the c-direction and in ab plane, correspondingly) linear corrections to the logarithmic interaction become important. In extremely anisotropic high-T_c oxides, like Bi- and Tl-based materials, the characteristic length of the crossover from a 2D to a 3D behaviour should be of the order of a few hundred ångströms.

Another concern is dissipation of the vortex motion. As has been mentioned in the introduction, at low temperature high-T_c materials are likely to enter the superclean regime where friction is small compared to other forces. This corresponds to the limit $\gamma \ll 1$ which is needed for the small width of the levels. The typical value of the sheet concentration of electrons in CuO_2 layers is $n_s d \approx 10^{14}$ cm^{-2}, which gives l of the order of a few Å.

Correspondingly, the typical value of the energy constant E_0 in equation (42) must be of the order of 10^3 K. Reasonable values of $m \gg 1$ are $m \sim 50$. This should correspond to the size of the orbit, $r = l\sqrt{2m}$, below 100Å, that is, greater than ξ but still in the range of the logarithmic interaction. For such m the spacing between the levels must be of the order of a few kelvin. The temperature, of course, should be much lower, to suppress thermal transitions between the levels. The lifetime of these levels should be of the order of $10^{-11}s/\gamma$, that is, about 10^{-9} s for $\gamma \sim 10^{-2}$. The levels of that kind fall in the microwave range of frequencies. The most direct way to observe them would be to study the absorption and radiation spectra of a superconductor in that range. The power of the radiation can be increased in the case of a massive annihilation of vortices and antivortices, produced by, e.g., the reversal of the magnetic field. One can also look for the discrete maxima in the noise spectrum of a superconductor.

In conclusion, we would like to note that the possibility of observing vortex levels is not at all obvious. Although we have presented heuristic arguments in favour of the narrow width of the levels in a superclean regime, the effect of the dissipation on quantum dynamics of vortices remains largely unknown. We believe, however, that such a possibility must be pointed out because of its fundamental importance for quantum physics.

Acknowledgments

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References

[1] Bonevich J E, Harada K, Matsuda T, Kasai H, Yoshida T, Pozzi G and Tonomura A 1993 *Phys. Rev. Lett.* **70** 2952
[2] Mitin A V 1987 *Zh. Eksp. Teor. Fiz.* **93** 590 (Engl. Transl. 1987 *Sov. Phys.–JETP* **66** 335)
[3] Motta A C, Visani P and Polini A 1988 *Phys. Rev.* **37** 9830
[4] Fruchter L, Malozemoff A P, Campbell I A, Sanchez J, Konczykowski M, Grissen R and Holtzberg F 1991 *Phys. Rev.* B **43** 8709
[5] Tejada J, Chudnovsky E M and Garcia A 1993 *Phys. Rev.* B **47** 11552
[6] Seidler G T, Carrillo C S, Rosenbaum T F, Welp U, Crabtree G W and Vinokur V M 1993 *Phys. Rev. Lett.* **70** 2814
[7] Prost D, Fruchter L, Campbell I A, Motohira N and Konczykowski M 1993 *Phys. Rev.* B **47** 3457
[8] Bulaevskii L N and Maley M P 1993 *Phys. Rev. Lett.* **71** 3541
[9] Bulaevskii L N, Cho J H, Maley M P, Kes P, Li Qiang, Suenaga M and Ledvij M 1994 *Phys. Rev.* B **50** 3507
[10] Blatter G, Geshkenbein V B and Vinokur V M 1991 *Phys. Rev. Lett.* **66** 3297
Blatter G and Geshkenbein V 1993 *Phys. Rev.* **47** 2725
[11] Ivlev B I, Ovchinnikov Yu N and Thompson P S 1991 *Phys. Rev.* B **44** 7023
[12] Feigelman M, Geshkenbein V B, Larkin A I and Levit A 1993 *Pis. Zh. Eksp. Teor. Fiz.* (Engl. Transl. **57** 699 1993 *JETP Lett.* **57** 711)
[13] Ao P and Thouless D J 1994 *Phys. Rev. Lett.* **72** 132
[14] Stephen M J 1994 *Phys. Rev. Lett.* **72** 1534
[15] Chudnovsky E M, Ferrera A and Vilenkin A 1995 *Phys. Rev.* B **51** 1181
[16] Kopnin N B and Kravtsov V E 1976 *Pis. Zh. Eksp. Teor. Fiz.* (Engl. Transl. **23** 631 1976 *JETP Lett.* **23** 578); 1976 *Zh. Eksp. Teor. Fiz.* **71** 1644 (Engl. Transl. 1976 *Sov. Phys.–JETP* **44** 861)
[17] Harris J M, Yan Y F, Tsui O K C, Matsuda Y and Ong N P 1994 *Phys. Rev. Lett.* **73** 1711
[18] Matsuda Y, Ong N P, Yan Y F, Harris J M and Peterson J B 1994 *Phys. Rev.* B **49** 4380

[19] de Gennes P G and Matricon J 1964 *Rev. Mod. Phys.* **36** 45
[20] Noziéres P and Vinen F 1966 *Phil. Mag.* **14** 667
[21] Davis R L 1990 *Mod. Phys. Lett.* A 5 853
[22] Ao P and Thouless D J 1993 *Phys. Rev. Lett.* **70** 2158
[23] Girvin S M and Jach T 1984 *Phys. Rev.* B **29** 5617
[24] Cataudella V and Minnhagen P 1990 *Physica* **166** 442
Minnhagen P and Olsson P 1991 *Phys. Rev.* B **44** 4503; 1991 *Phys. Rev. Lett.* **67** 1039
Minnhagen P 1992 *Helv. Phys. Acta* **65** 205
Minnhagen P and Olsson P 1992 *Phys. Rev.* B **45** 5722
[25] Glazman L I and Koshelev A E 1990 *Zh. Eksp. Teor. Fiz.* **97** 1371 (Engl. Transl. 1990 *Sov. Phys.–JETP* **70** 774)
[26] Artemenko S N and Kruglov A N 1990 *Phys. Lett.* **143A** 485
[27] Bulaevskii L N, Ledvij M and Kogan V G 1992 *Phys. Rev.* B **46** 366
[28] Chattopadhyay B and Shenoy S R 1994 *Phys. Rev. Lett.* **72** 400